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Hypothetical Solution of the Problem of Measurement Through the Notion of Quantum Backward Causality

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The notion of the *coexistence* in the quantum framework of causality forward or backward in time is proposed as a possible way for solving the problem of measurement.

We shall explore the possibility of the coexistence in the quantum framework of the mutually exclusive alternatives of forward or backward causality, a notion which appears to be closely related to the problem of measurement. It should be anticipated that this work is preliminary in character, and is based on analogies, ad hoc assumptions, and on the consideration of correlation or symmetry, *not* on a dynamical view.

We shall avail ourselves of the interplay between *basic* physical considerations and logical/computational ones. Let C be a computer with input/output S_1, S_2 ranging over $+$, $-$. Either S_1 or S_2 can be the input; the other correspondingly is the output. Causality goes of course from input to output, thus from S_1 to S_2, *or* (mutually exclusive here and in the following) from S_2 to S_1. The *complete description* of a possible behavior of C is (I) $S_1 = + (-)$ *causes* $S_2 = - (+)$ *or* $S_2 = - (+)$ *causes* $S_1 = + (-)$. If it is not stated outside of statement (I) that C is logically reversible, the second part of (I) (after the *or*) is not redundant, e.g., it could be instead $S_2 = - (+)$ *causes* $S_1 = - (+)$. We assume that statement (I), as it is, can be applied to mutual causality between the eigenstates of compatible attributes of a steady state, a notion from quantum steady computation (Castagnoli and Rasetti, 1993), and to causality between past and future of the evolution of

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a closed system which undergoes state vector reduction [from the preparation $|\psi(t_1)\rangle$ to the state at the outcome of measurement $|\psi(t_2)\rangle$], as follows. Let $|\psi^c(t_1)\rangle$ and $|\psi^c(t_2)\rangle$, with $c = f, b$, be independent ket variables spanning the possible system states at times t_1 and $t_2 > t_1$: (II) *For every* $|\psi^f(t_1)\rangle$ *and* $|\psi^b(t_2)\rangle$, $|\psi^f(t_1)\rangle$ *causes* $|\psi^f(t_2)\rangle = U^f|\psi^f(t_1)\rangle$ *or* $|\psi^b(t_2)\rangle$ *causes* $|\psi^b(t_1)\rangle = (U^b)^{-1}|\psi^b(t_2)\rangle$, where, *up to a phase factor* (as clarified in below), U^f or U^b is the unitary transformation undergone by the *deterministic* (without state vector reduction) evolution from t_1 to t_2. We assume that the second part of (II) is not redundant since, on one hand, a process which undergoes state vector reduction is not reversible in character, and on the other hand we require that, *for each individual evolution*, in the time-reversed picture the system (up to overall phase) reruns backward the same evolution it ran forward [which is what (II) says]. For example, omitting the second part of (II) brings in the paradox (Penrose, 1986) that a photon reflected by a half-silvered mirror could in the time-reversed picture be transmitted in contradiction with its previous history. We also assume that the two parts of statement (II) [or (I)], which are logically mutually exclusive, are mapped onto a pair of components of a quantum superposition which maps the complete statement (and will reconcile the initial state with the final one after reduction). The foregoing will be applied to an entangled state, then applied by analogy to temporal causality.

Given the spin singlet state $|\psi\rangle = (1/\sqrt{2})(|+\rangle_1|-\rangle_2 - |-\rangle_1|+\rangle_2)e^{i\omega t}$, S_1 and S_2 play the role of the spin eigenvalues of particles 1 and 2. We assume that the complete statement (I) corresponds to the completely defined (pure) state $|\psi\rangle$ [causal implications between the eigenstates of compatible attributes of a steady state are outside time and do not need to be logically reversible, they can map irreversible gates (Castagnoli and Rasetti, 1993)]. Let θ be the angle between the direction of particle 1 spin and that of spin $+$, and $H_i = \text{span}\{|+\rangle_i, |-\rangle_i\}$ the Hilbert space of the spin of particle i. We introduce $|\psi\rangle_1 = \cos\phi\,|+\rangle_1 + \sin\phi\,|-\rangle_1$, $|\psi\rangle_2 = -\sin\phi\,|+\rangle_2 + \cos\phi\,|-\rangle_2$ as the *stochastic descriptions* of the individual spin states of particles 1 and 2, with $\phi = \theta/2$ a stochastic variable with uniform distribution in $[0, 2\pi]$. θ is a hidden *stochastic* variable. Averaging over ϕ brings us from the stochastic to the conventional representation: $\langle|\Psi\rangle_i\rangle_\phi = 0$ means the absence of a (defined) spin state of particle i, whereas $\langle|\Psi\rangle_i\langle\Psi|\rangle_\phi = \frac{1}{2}(|+\rangle_i\langle+|+|-\rangle_i\langle-|)$. Thus $|\Psi\rangle_i$ stochastically represents the mixture, with even weights, of all possible spin directions of particle i. We can represent $|\Psi\rangle_1$ ($|\Psi\rangle_2$) in $H = H_1 \otimes H_2$ by using the causal implications of the first (second) part of (I):

$$|\Psi\rangle_1 \to e^{i\delta^{1,2}}|\Psi^{1,2}\rangle = e^{i\delta^{1,2}}(\cos\phi|+\rangle_1|-\rangle_2 + \sin\phi\,|-\rangle_1|+\rangle_2)$$

$$|\Psi\rangle_2 \to e^{i\delta^{2,1}}|\Psi^{2,1}\rangle = e^{i\delta^{2,1}}(\cos\phi\,|+\rangle_1|-\rangle_2 - \sin\phi\,|-\rangle_1|+\rangle_2)$$

We assume that (i) $|\Psi^{1,2}\rangle$ ($|\Psi^{2,1}\rangle$) is the stochastic representation of the state of both particles corresponding to the first (second) part of (I); thus to one-way causality from 1 to 2 (2 to 1); (ii) both states coexist in superposition for a suitable definition of $\delta^{1,2}$ and $\delta^{2,1}$; and (iii) such a superposition maps the *complete* statement (I) and is therefore the stochastic description of $|\Psi\rangle$:

$$|\Psi\rangle = \langle pe^{i\delta_{1,2}}|\Psi\rangle^{1,2} + pe^{i\delta_{2,1}}|\Psi\rangle^{2,1}\rangle_{\phi}$$

The p's are weights identical because of symmetry. This yields $p = 1$, $\delta^{1,2} = \phi$, $\delta^{2,1} = \phi \pm \pi/2$ (up to a rotation of all phases) and the following consequences: First,

$$|\langle e^{i\phi}|\Psi\rangle^{1,2}\rangle_{\phi}|^2 + |\langle e^{i(\phi\pm\pi/2)}|\Psi\rangle^{2,1}\rangle_{\phi}|^2 = 1$$

Second, by permuting the two particles and changing ϕ into $\phi \pm \pi/2$ (which is stochastically equivalent), the stochastic states $e^{i\phi}|\Psi\rangle^{1,2}$ and $e^{i(\phi\pm\pi/2)}|\Psi\rangle^{2,1}$ change into one another, are therefore stochastically indistinguishable. This is in agreement with the fact that (I) [like (II)] induces a recursion: $S_1 = +\ (-)$ *causes* $S_2 = -\ (+)$, *which causes* $S_1 = +\ (-)$, etc. Causality from 1 to 2 or 2 to 1 is not distinguishable, is *mutual*. By using such "consequences" as "conditions" instead, $|\Psi\rangle_i$ can be introduced as the generic vector of H_i. Applying the relevant conditions brings us from "generic" to "stochastically defined."

Going to *mutual causality in time evolutions*, we do not have to introduce a stochastic description of $|\psi(t_1)\rangle$ and $|\psi(t_2)\rangle$, which are pure states. Stochastic character will appear at the level of the components into which the complete evolution $|\psi(t)\rangle$ [corresponding to (II)] will be decomposed: a *forward* (*backward*) evolution associated with causality from t_1 to t_2 (t_2 to t_1) and corresponding to the first (second) part of (II). The former equations become

$$|\psi(t)\rangle = e^{i\delta_t^f}|\psi^f(t)\rangle + e^{i\delta_t^b}|\psi^b(t)\rangle, \qquad ||\psi^f(t)\rangle|^2 + ||\psi^b(t)\rangle|^2 = 1 \qquad (1)$$

The average over ϕ is dropped and weights are incorporated into the evolutions. The latter equation should represent the mutual exclusivity of the two parts of (II). Assumptions/rules are as follows: (a) $|\psi^f(t)\rangle$ and $|\psi^b(t)\rangle$ are introduced with generic initial (or final) amplitudes (on the basis vectors of H) and follow the transformations of the deterministic evolution, which, freed from the initial condition [to be applied to $|\psi(t)\rangle$], is taken as a model of one-way causality. Consequently $F^2 = ||\psi^f(t)\rangle|^2$ and $B^2 = ||\psi^b(t)\rangle|^2$ remain constant along t; (b) for every $t_1 \neq t_2$, $\delta^f_{t_1}$, $\delta^f_{t_2}$, $\delta^b_{t_1}$, and $\delta^b_{t_2}$ are introduced as free independent phases; thus, if U is a transformation of the conventional evolution, the corresponding transformation U^f (U^b) of the forward (backward) evolution is

$$U^f = e^{i\delta^f}U \quad (U^b = e^{i\delta^b}U)$$

where δ^f and δ^b are independent of one another; (c) δ^f and δ^b are independent of the initial amplitudes of the corresponding evolutions—due to the linearity of transformations; (d) $[\exp(i\delta_t^f)|\psi^f(t)\rangle$ and $[\exp(i\delta_t^b)]|\psi^b(t)\rangle$ are stochastically indistinguishable; (e) normalization, *both* the initial and final conditions are applied to the complete evolution $|\psi(t)\rangle$. After the application of all the relevant conditions, the free independent variables will become correlated stochastic variables.

We shall exemplify the application of the model in question. Consider a spin-1/2 particle emerging from a spin filter rotated by φ with respect to a Stern–Gerlach device (SG) with detectors A and B at its two outputs. By denoting $|S\rangle_Y$ the tensor product $|\Phi_Y(\mathbf{x})\rangle|S\rangle$, where $|\Phi_Y(\mathbf{x})\rangle$ are the normalized wave functions localized in the disjoint regions Y = input I, detector A, and detector B, and $|S\rangle$ is the spin ket, the states at the input I and at the output O are conventionally

$$\begin{aligned}|\psi^c\rangle_\mathrm{I} &= \cos\varphi\,|+\rangle_\mathrm{I} + \sin\varphi\,|-\rangle_\mathrm{I} \to |\psi^c\rangle_\mathrm{O}\\ &= \eta_+\cos\varphi\,|+\rangle_\mathrm{A} + \eta_-\sin\varphi\,|-\rangle_\mathrm{B}\end{aligned}$$

where η_+ and η_- are phase factors. Such deterministic input–output "evolution" is substituted by the couple of evolutions

$$|\psi^f\rangle_\mathrm{I} = f_+|+\rangle_\mathrm{I} + f_-|-\rangle_\mathrm{I} \to |\psi^f\rangle_\mathrm{O} = \eta^f(\eta_+f_+|+\rangle_\mathrm{A} + \eta_-f_-|-\rangle_\mathrm{B})$$

$$|\psi^b\rangle_\mathrm{I} = b_+|+\rangle_\mathrm{I} + b_-|-\rangle_\mathrm{I} \to |\psi^b\rangle_\mathrm{O} = \eta^b(\eta_+b_+|+\rangle_\mathrm{A} + \eta_-b_-|-\rangle_\mathrm{B})$$

where $f_\pm$, $b_\pm$ are "generic" amplitudes, $\eta^f = \exp(i\delta^f)$ and $\eta^b = \exp(i\delta^b)$. The complete evolution is $|\psi\rangle_\mathrm{I} = |\psi^f\rangle_\mathrm{I} + |\psi^b\rangle_\mathrm{I} \to |\psi\rangle_\mathrm{O} = |\psi^f\rangle_\mathrm{O} + |\psi^b\rangle_\mathrm{O}$. The second of equations (1) on the forward/backward evolutions and normalization on the complete evolution yield

$$\begin{aligned}&\cos(\gamma_+^f - \gamma_+^b)/\cos(\gamma_+^f - \gamma_+^b + \delta^f - \delta^b)\\ &= \cos(\gamma_-^f - \gamma_-^b)/\cos(\gamma_-^f - \gamma_-^b + \delta^f - \delta^b)\end{aligned}$$

where $\gamma_\pm^f$ and $\gamma_\pm^b$ are the phases of $f_\pm$ and $b_\pm$. Rule (c) requires that the above equation is an identity, yielding $\delta^f = \delta^b$ ($U^f = U^b$) or $\delta^f = \delta^b \pm \pi$ ($U^f = -U^b$).

Now, $\varphi \neq 0$ (or $\varphi \neq \pi$) requires $\delta^f = -\delta^b$. Assuming the final condition that A clicks yields $f_+ = \frac{1}{2}(\cos\varphi + \eta)$, $b_+ = \frac{1}{2}(\cos\varphi - \eta)$, $f_- = b_- = \frac{1}{2}\sin\varphi$, where $\eta = e^{i\delta}$ and δ is a free phase,

$$|\psi\rangle_\mathrm{I} = \cos\varphi\,|+\rangle_\mathrm{I} + \sin\varphi\,|-\rangle_\mathrm{I} \to |\psi\rangle_\mathrm{O} = \eta'|+\rangle$$

where $\eta' = \eta\eta^f\eta_+$, $F^2 = \frac{1}{2} + \frac{1}{2}\cos\varphi\cos\delta$, $B^2 = \frac{1}{2} - \frac{1}{2}\cos\varphi\cos\delta$. The condition $\varphi \neq 0$ implies $F^2, B^2 \neq 0$ and indistinguishability requires δ to be a stochastic variable with uniform distribution in $[0, 2\pi]$. The forward and backward evolutions acquire a stochastic character. State vector reduction (or equivalent thereof) is localized in the region where the SG device performs the above input–output transformation.

The condition $\varphi = 0$ allows either $\delta^f = -\delta^b$ or $\delta^f = \delta^b$. There can be either just one evolution (forward or backward) or both with F^2, B^2 stochastic as in the previous case with $\varphi = 0$.

By using as a final condition the *generic* spin direction φ', the extremal values of F^2 or B^2 become $\frac{1}{2} \pm \frac{1}{2}\cos(\varphi - \varphi')$. No measurement (or ignoring it) implies no backward (thus no mutual) causality, hence $\varphi = \varphi'$, namely the deterministic evolution.

We shall outline other implications. A half-silvered mirror followed by two photon detectors requires $\delta^f = -\delta^b$ and mutual causality, which prevents the aforementioned paradox. Measurement correlations in, say, spin singlet states can be modeled on the basis that the spin direction determined by the first measurement causes backward in time the (opposite) spin direction of the other particle. Mutual definition between the quantum process and the result of measurement of Bohr's interpretation, by changing "definition" into "causality," becomes causality from process to result—forward in time—or from result to process—backward in time. Mutual causality, when the process is a computational one and the result is the solution of a problem, appears to be at the heart of the notion of nondeterministic computation in both quantum steady computation (Castagnoli, 1991; Castagnoli *et al.*, 1992; Castagnoli and Rasetti, 1993) and quantum parallel computation (Deutsch *et al.*, 1992; Shor, 1994). That causality along closed timelike lines might yield nondeterministic computation appears in Deutsch (1991). Implementing such a form of computation might be the way to verify the existence of mutual causality.

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